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2003 J. Phys. A: Math. Gen. 36 9371

(http://iopscience.iop.org/0305-4470/36/35/321)

INSTITUTE OF PHYSICS PUBLISHING JOURNAL OF PHYSICS A: MATHEMATICAL AND GENERAL

J. Phys. A: Math. Gen. **36** (2003) 9371–9378 PII: S0305-4470(03)63829-1

Quantized massive collective excitations, short-range spin fluctuations and high-T_c superconductivity

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Received 23 May 2003
Published 20 August 2003
Online at stacks.iop.org/JPhysA/36/9371

Abstract

We have succeeded in quantizing massive collective gauge fields around the doped hole by using the theoretical formula, which is based on the gauge-invariant effective Lagrangian density, in underdoped cuprates. Quantized massive gauge fields, which are induced around the doped hole, explain naturally the broad spectra of angle-resolved photoemission around the hot spot, short-range spin fluctuations and anomalous transport properties in underdoped cuprates.

PACS numbers: 75.10.Nr, 75.10.Jm, 74.70.Vy

1. Introduction

The anomalous properties in underdoped cuprates remain controversial subjects. Angle-resolved photoemission (ARPES) in underdoped cuprates is quite unusual [1]. For momentum $(\pi, 0)$ or $(0, \pi)$, the spectral function of ARPES is anomalously broad, which means strong scattering. Thus photoemission implies a lifetime that is generally short but has a pronounced angular dependence. The nuclear magnetic or nuclear quadrupole resonance (NMR or NQR) experiments for underdoped cuprates show that the nuclear spin relaxation rate, $1/T_1T$, has a Curie-like $1/T$ dependence at high T, generally associated with antiferromagnetic correlations, but then falls at low T due to the opening of the pseudogap [2]. From the NMR/NQR experiments, Tokunaga *et al* [3] have found a very important relation. That is, the superconducting gap and the characteristic energy Γ of the spin fluctuation are correlated with each other.

The normal-state transport properties in underdoped cuprates are very anomalous. Especially the anisotropic scattering of the hole in Cu–O planes, T-linear resistivity and the anomalous c-axis transport are very important subjects. Various models, which might explain the anisotropic scattering property of the hole in Cu–O planes, have been proposed [4–6].

0305-4470/03/359371+08$30.00

A central concept is the hot spot near $(\pi, 0)$ and $(0, \pi)$. In these models the hole lifetime on most of the Fermi surface is much longer than in the hot spot. Recently the present author has succeeded in quantizing the massive collective modes, whose mass depends strongly on the angle of Fermi momentum, $\hat{k}_{\rm F}$, of the hole on the Fermi surface in the gauge-invariant formula [7–9].

In this paper, we give some explanations for the anomalous properties in underdoped cuprates, using the theoretical formula, which is based on the gauge-invariant effective Lagrangian density in the quasi-two-dimensional correlated electron system.

2. Quantized massive collective gauge fields around the doped carrier

When holes are doped lightly in a quasi-(2 + 1)-dimensional quantum antiferromagnet, distortion appears around the doped hole. Since the distortion around the hole is due to strong many-body effects, the distortion effects around the hole are nonlinear (Yang–Mills field-like). Taking into account that the symmetry in the undoped (2 + 1)-dimensional quantum antiferromagnet is invariant under local $SU(2)$ [10], we think that the distorting gauge fields A^a_μ (Yang–Mills fields) introduced by the hole have a local $SU(2)$ symmetry. Since the distortion field is short-range-like, the parts of the gauge fields are massive. Thus it is assumed that $SU(2)$ gauge fields A^a_μ are spontaneously broken through the Anderson–Higgs mechanism in a way similar to the breaking of the antiferromagnetic symmetry around the hole.

We set the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0, 0, \mu(\hat{k}_{\rm F})\rangle$ of the Bose field ϕ_a in the Lagrangian density as follows,

$$\begin{aligned} L = {} & \frac{1}{2}\left(\partial_i N^j_c - g_1\varepsilon_{abc}\varepsilon_{ijk}A^b_i N^k_a\right)^2 + \psi^+\left(\mathrm{i}\partial_0 - g_2 T_a A^a_0\right)\psi - \frac{1}{2m}\psi^+\left(\mathrm{i}\nabla - g_2 T_a A^a_{(\mu\neq 0)}\right)\psi \\ & - \frac{1}{4}\left(\partial_\nu A^a_\mu - \partial_\mu A^a_\nu + g_3\varepsilon_{abc}A^b_\mu A^c_\nu\right)^2 + \frac{1}{2}\left(\partial_\mu\phi_a - g_4\varepsilon_{abc}A^b_\mu\phi_c\right)^2 \\ & - \lambda^2(\phi_a\phi_a - \mu^2)^2. \end{aligned} \qquad (1)$$

$\hat{k}_{\rm F}$ is the vector of the Fermi momentum. After the symmetry breaking $\langle 0|\phi_a|0\rangle = \langle 0, 0, \mu(\hat{k}_{\rm F})\rangle$, that is, transition of fields ϕ_a with a zero asymptotics at infinity,

$$(\phi_1, \phi_2, \phi_3) \longrightarrow (\phi_1, \phi_2, \mu(\hat{k}_{\rm F}) + \phi_3)$$

makes the isotopic symmetry breaking explicit, and we can obtain the effective Lagrangian density, $\mathcal{L}_{\rm eff}$, at small doping of holes [11, 12]. That is, $\langle 0|\phi_3|0\rangle$ can be regarded as a kind of disorder parameter [13]. The value, $\mu(\hat{k}_{\rm F})$, of the symmetry breaking depends strongly on the direction of the Fermi momentum, $\hat{k}_{\rm F}$, on the Fermi surface. Furthermore, the value $\mu(\hat{k}_{\rm F})$, is much correlated with the gap energy of the high-energy pseudogap. If the value of the high-energy pseudogap is related to the strength of the antiferromagnetic short-range order [14], the distortion, which is induced by the doped hole, becomes larger as the hole is doped in the state of the larger gap energy of the high-energy pseudogap. In addition, since the gap energy of the high-energy pseudogap is rapidly reduced with the increase of the doping level p, the value, $\mu(\hat{k}_{\rm F})$, is reduced with the increase of the doping level p. Taking account of the value, $\mu(\hat{k}_{\rm F})$, means the strength of the distortion induced by the doped hole, the value $\mu(\hat{k}_{\rm F})$, is higher around the hot spot,

$$\begin{aligned}\mathcal{L}_{\rm eff} = \frac{1}{2}\left(\partial_i N_c^j - g_1\varepsilon_{abc}\varepsilon_{jik}A_i^b N_a^k\right)^2 + \psi^+\left(\mathrm{i}\partial_0 - g_2 T_a A_0^a\right)\psi \\ - \frac{1}{2m}\psi^+\left(\mathrm{i}\nabla - g_2 T_a A^a_{(\mu\neq 0)}\right)^2\psi - \frac{1}{4}\left(\partial_\nu A_\mu^a - \partial_\mu A_\nu^a + g_3\varepsilon_{abc}A_\mu^b A_\nu^c\right)^2 \\ + \frac{1}{2}\left(\partial_\mu\phi_a - g_4\varepsilon_{abc}A_\mu^b\phi_c\right)^2 + \frac{1}{2}m_1^2\left[\left(A_\mu^1\right)^2 + \left(A_\mu^2\right)^2\right] \\ + m_1\left[A_\mu^1\partial_\mu\phi_2 - A_\mu^2\partial_\mu\phi_1\right] + g_4 m_1\left\{\phi_3\left[\left(A_\mu^1\right)^2 + \left(A_\mu^2\right)^2\right]\right. \\ \left. - A_\mu^3\left[\phi_1 A_\mu^1 + \phi_2 A_\mu^2\right]\right\} - \frac{m_2^2}{2}(\phi_3)^2 - \frac{m_2^2 g_4}{2m_1}\phi_3(\phi_a)^2 - \frac{m_2^2 g_4^2}{8m_1^2}(\phi_a\phi_a)^2 \qquad (2)\end{aligned}$$

where N_a^i is the spin parameter, ψ is the Fermi field of the hole, $m_1 = \mu \cdot g_4$, $m_2 = 2\sqrt{2}\lambda \cdot \mu$, and T_a are the $SU(2)$ generators. The effective Lagrangian describes two massive vector fields A_μ^1 and A_μ^2, and one massless $U(1)$ gauge field A_μ^3. Although we have explicitly broken isotopic invariance, the effective Lagrangian density is invariant under local gauge transformations, $\omega(r) = 1 + \alpha^a(r)T_a$, tending to unity at infinity. We shall give the explicit form of the gauge transformations in new variables, confining ourselves to infinitesimal transformations, $\delta\phi_a = -g_4\varepsilon_{abc}\phi_b\alpha^c - m_1\varepsilon_{a3c}\alpha^c$.

Extending the previous theory [11, 12], the generation function $Z[J]$ for the Green functions is introduced as follows,

$$Z[J] = \int \mathcal{D}A\mathcal{D}B\mathcal{D}N\mathcal{D}C\mathcal{D}\bar{C}\mathcal{D}\psi^+\mathcal{D}\psi\mathcal{D}\phi \exp \mathrm{i}\int \mathrm{d}^4x\,(\mathcal{L}_{\rm eff} + \mathcal{L}_{\rm GF+FP} + J\cdot\Phi) \qquad (3)$$

$$\mathcal{L}_{\rm DF+FP} = B^a\partial^\mu A_\mu^a + \tfrac{1}{2}\alpha B^a B^a + \mathrm{i}\bar{C}^a\partial^\mu\mathcal{D}_\mu C^a \qquad (4)$$

where B^a and C^a are Nakanishi–Lautrup (NL) fields and Faddeev–Popov fictitious fields, respectively,

$$J\cdot\Phi \equiv J^{a\mu}A_\mu^a + J_B^a B^a + J_N^a\cdot N_a + \bar{J}_C^a\cdot C^a + J_{\bar{C}}^a\bar{C}^a + \bar{\eta}\psi + \eta\psi^+ + J_\phi^a\phi_a. \qquad (5)$$

BRS-quartets [15, 16] in the present theoretical system are $(\phi_1, B^1, C^1, \bar{C}^1)$, $(\phi_2, B^2, C^2, \bar{C}^2)$ and $\left(A_{L,\mu}^3, B^3, C^3, \bar{C}^3\right)$, where $A_{L,\mu}^3$ is the longitudinal component of A_μ^3. So we need these fields for the unitarity condition, although these fields are unobservable and fictitious ones. It is fascinating that a part of the present theoretical formula is similar to the so-called Georgi–Glashow model for unifying weak and electromagnetic interactions [17].

Because the masses of A_μ^1 and A_μ^2 are formed through the Higgs mechanism by introducing the hole, the fields A_μ^1 amd A_μ^2 exist around the hole within a length of $\sim 1/m_1 \equiv R_c$.

The quantized gauge fields A_μ^a are expressed as

$$A_\mu^a = (2\pi)^{-3/2}\int\left[a^a(p)e_\mu^a(p)\exp(\mathrm{i}pr) + a^{a+}(p)e_\mu^a(p)\exp(-\mathrm{i}pr)\right]\mathrm{d}^3p\Big/\sqrt{2\omega_p^a}$$

where $\omega_p^a = \sqrt{p^2 + m_1^2}\,(a = 1, 2)$ and $\omega_p^a = \sqrt{p^2}\,(a = 3)$, $a^{a+}(p)$ and $a^a(p)$ are the creation and annihilation operators of the gauge particle A_μ^a with momentum p, respectively, and $e_\mu^a(p)$ are the polarization vectors. The masses, m_1, of the gauge fields A_μ^1 and A_μ^2 induced by the hole depend strongly on the angle of Fermi momentum of the hole on the Fermi surface. The value, m_1, is higher in the case of the hole around the hot spot. It is thought that the interaction between two holes for the Cooper pair formation is derived from the exchange of the fields

A_μ^a [11, 12, 18–20]. The effective interaction between two holes at $k \sim \left(-\hat{k}_F^h, 0\right)$ and $k \sim \left(\hat{k}_F^h, 0\right)$ for the Cooper pair formation is given approximately as

$$H_{\text{int}} \sim {g_2}^2 \frac{\left(\omega_{2\hat{k}_F^h}\right)^2}{\left(\varepsilon_{\hat{k}_F^h} - \varepsilon_{-\hat{k}_F^h}\right) - \left(\omega_{2\hat{k}_F^h}\right)^2} C^+_{\hat{k}_F^h} C_{-\hat{k}_F^h} C^+_{-\hat{k}_F^h} C_{\hat{k}_F^h}$$

where $\hat{k}_F^h$ is the Fermi momentum at the hot spot, and $C^+_{\hat{k}_F^h}$ and $C_{\hat{k}_F^h}$ are the creation and annihilation operators of the hole with momentum $\hat{k}_F^h$, respectively,

$$\omega_{2\hat{k}_F^h} = \sqrt{(2k_F)^2 + {m_1}^2} = \sqrt{(2k_F)^2 + \left(\mu\left(\hat{k}_F^h\right)g_4\right)^2}.$$

The value of m_1 becomes higher around the hot spot. This means that the attractive interactions between two holes at $k \sim \left(-\hat{k}_F^h, 0\right)$ and $k \sim \left(\hat{k}_F^h, 0\right)$ or between two holes at $k \sim \left(0, -\hat{k}_F^h\right)$ and $k \sim \left(0, \hat{k}_F^h\right)$ for the Cooper pair formation are the strongest ones in comparison with those of the other pairs. As described previously [11, 12], the effective interaction between two holes is like the asymptotic freedom, since the *SU*(2) Yang–Mills fields are more effective when the distance between the holes is shorter than $\sim 2/m_1$.

The present theory is based on the local *SU*(2) formulation [11, 12]. Although each standpoint is very different, a part of the present formula is similar to the recent *SU*(2) spinon–holon model [21, 22]. In addition, the present theory satisfies the gauge-invariance, renormalization and unitarity conditions.

3. Angle-resolved photoemission spectra in underdoped cuprates

The interesting results of the $(\pi, 0)$ photoemission spectra, whose features are very broad, for insulating Ca_2CuO_2Cl and Dy-doped $Bi_2Sr_2CaCu_2O_{8+\sigma}$ stress that the $(\pi, 0)$ spectra evolve continuously from the insulator to metal [23]. In particular, it means that the 100–200 meV high-energy pseudogap in underdoped cuprates can be connected to the property of an insulator as pointed out by Laughlin [23]. This suggests strongly that the mechanism of the $(\pi, 0)$ broad spectra must be derived from some parameters connected with the high-energy pseudogap. It should be noted that the distortion strength, which is identified with the symmetry breaking $\mu(k_F)$ in the mass $= \mu(k_F)g_4$ of the massive gauge fields around the hole, is connected with the high-energy pseudogap.

Now we shall consider the broad feature of the ARPES near $(\pi, 0)$ in underdoped high-T_c cuprates from the present theoretical view. The creation of a photohole is more likely to produce collective excitation plus a hole in the quasiparticle band. From equation (3), we can obtain the Green functions of the massive gauge fields A_μ^1 and A_μ^2 in the 't Hooft–Feynman gauge as follows, that is, the Fourier transform of $\left\langle A_\mu^a A_\nu^a \right\rangle_{a=1,2}$ is $D_R(k, \omega) \sim \left\{g_{\nu\mu} \big/ \left(\omega^2 - \left(k^2 + m_1^2\right) + \Pi\right)\right\}$. The hole spectral function is represented by

$$A(p, \varepsilon) \propto \frac{\mathrm{Im}\Sigma(p, \varepsilon)}{\left[\varepsilon - \varepsilon_k - \mathrm{Re}\Sigma(p, \varepsilon)\right]^2 + \left[\mathrm{Im}\Sigma(p, \varepsilon)\right]^2} \tag{6}$$

where the self-energy is given by

$$\Sigma(p, \varepsilon) = -\frac{{g_2}^2}{(2\pi)^2\pi} \int \mathrm{d}p_1 \int_{-\infty}^{\infty} \mathrm{d}\omega \int_{-\infty}^{\infty} \mathrm{d}\varepsilon_1 \frac{\mathrm{Im}\, D_R(p - p_1, \omega) A(p_1, \varepsilon_1)}{\omega + \varepsilon_1 - \varepsilon - \mathrm{i}\delta} \times \left(\tanh\frac{\varepsilon_1}{2T} + \coth\frac{\omega}{2T}\right). \tag{7}$$

The recoil relaxation with collective excitations occurs in a higher-energy region in comparison with the hole energy $\varepsilon_k + \mathrm{Re}\Sigma(p, \varepsilon)$.

Thus, in the case of $\varepsilon \gg \sqrt{m_1^2 + (2k_F)^2}$, the imaginary part of the self-energy is introduced approximately as follows,

$$\mathrm{Im} \sum(\varepsilon) \sim \sqrt{m_1^2 + (2k_F)^2}.$$

This means that, in a higher-energy region, the recoil relaxation is dominantly due to emission and absorption of the massive gauge fields A_μ^1 and A_μ^2. As described before, it should be noted that the value of the mass, m_1, of the massive gauge fields A_μ^1 and A_μ^2 is higher around the hot spot. So the present theory predicts that the broad feature of ARPES line shape is remarkable near the hot spot. This is consistent with the experimental results [24, 25].

Also, as the doping increases, both gap energies of the low- and high-energy pseudogaps decrease remarkably, and these pseudogaps annihilate in the overdoped region. On the basis of the decrease of the gap energy of the high-energy pseudogap, the value $\mu(k_F)$ of the symmetry breaking and the masses of the gauge fields A_μ^1 and A_μ^2 decrease as the doping increases, and the massive gauge fields almost annihilate in the overdoped region. Thus, the value of the imaginary part of the self-energy decreases as the doping increases, and the broad feature in the ARPES spectrum annihilates in the overdoped region.

4. Short-range spin fluctuation induced by the massive gauge fields

We shall consider an important relation [3] between the superconducting gap and the characteristic energy Γ of the spin fluctuation from the present theoretical view. The NMR or NQR technique has supplied microscopic information on the doped high-T_c copper oxides. The nuclear spin-lattice relaxation rate is given by

$$1/T_1 T \sim \gamma_N^2 A_{\mathrm{hf}}^2 \sum_q \mathrm{Im}\, \chi^{-+}(q,\omega)/\omega$$

where ω is the nuclear resonance frequency, γ_N the gyromagnetic ratio of the nuclear spin and A_{hf} the hyperfine coupling constant. $\mathrm{Im}\, \chi^{-+}(q,\omega)$ is the imaginary part of the dynamic spin susceptibility. In underdoped high-T_c cuprates, it is seen from the first term in equation (2) that the massive gauge fields A_μ^1 and A_μ^2, which are introduced by the doped holes, strongly induce the short-range spin fluctuation.

Then we can estimate approximately the imaginary part of the dynamical spin susceptibility using equations (6) and (7) taking account of the effect of the massive gauge fields as follows,

$$\begin{aligned}\frac{\mathrm{Im}\, \chi^{-+}(q,\omega)}{\omega} &\sim \frac{1}{\omega}\mathrm{Im}\left(\mathrm{i}\int_0^\infty \mathrm{d}t\, \mathrm{e}^{\mathrm{i}\omega t}\left\langle\left[S_q^-(t), S_{-q}^+(0)\right]\right\rangle\right) \sim \int \delta(k-k_F)A(k+q,\varepsilon)\,\mathrm{d}k \\ &\propto \int \mathrm{d}k\,\delta(k-k_F)\frac{\mathrm{Im}\Sigma(k+q,\varepsilon_{k+q})}{\left(\varepsilon_{k+q}-\varepsilon_k-\mathrm{Re}\Sigma(k+q,\varepsilon_{k+q})\right)^2+\left(\mathrm{Im}\Sigma(k+q,\varepsilon_{k+q})\right)^2} \\ &\propto \frac{\mathrm{Im}\Sigma(k_F+q,\varepsilon_{k_F+q})}{\left(\varepsilon_{k_F+q}-\varepsilon_{k_F}-\mathrm{Re}\Sigma(k_F+q,\varepsilon_{k_F+q})\right)^2+\left(\mathrm{Im}\Sigma(k_F+q,\varepsilon_{k_F+q})\right)^2}.\end{aligned} \quad (8)$$

When $\mathrm{Re}\Sigma\left(k_F+q,\varepsilon_{k_F+q}\right)$ is small in comparison with $\varepsilon_{k_F+q}-\varepsilon_{k_F} \sim \omega$, equation (8) is represented as follows,

$$\frac{\mathrm{Im}\, \chi^{-+}(q,\omega)}{\omega} \sim \frac{\mathrm{Im}\Sigma(k_F+q,\varepsilon_{k_F+q})}{\omega^2+\left(\mathrm{Im}\Sigma(k_F+q,\varepsilon_{k_F+q})\right)^2} \quad (9)$$

where

$$\Sigma(k_{\rm F}+q,\varepsilon_{k_{\rm F}+q}) = -\frac{g_1^2}{(2\pi)^2\pi}\int {\rm d}p_1 \int_{-\infty}^{\infty}{\rm d}\omega\int_{-\infty}^{\infty}{\rm d}\varepsilon_1 \frac{{\rm Im}\, D_R(k_{\rm F}+q-P_1,\omega)A(P_1,\varepsilon_1)}{\omega+\varepsilon_1-\varepsilon_{k_{\rm F}+q}-{\rm i}\delta} \times\left(\tanh\frac{\varepsilon_1}{2T}+\coth\frac{\omega}{2T}\right). \quad (10)$$

$D_R\ (k_{\rm F}+q-P_1,\omega)$ is the Green function of the massive gauge fields, and $A(P_1,\varepsilon_1)$ corresponds to equation (6). Since ImΣ means the relaxation due to emission and absorption of the massive gauge fields A_μ^1 and A_μ^2, we can see that the characteristic energy Γ of the short-range spin fluctuation is attributed to ImΣ $\propto$ the mass, $m_1(\mu_{\hat{k}_{\rm F}})$, of the massive gauge fields A_μ^1 and A_μ^2 around the hot spot $(0,\pi)$. Because the superconducting gap around the hot spot $(0,\pi)$ is derived from the Cooper pair formation, which is introduced from exchange interaction of the massive gauge fields A_μ^1 and A_μ^2, we can introduce approximately the relation in which the superconducting gap is correlated with the mass, m_1, of the massive gauge field.

In other words, we can obtain the important relation in which the superconducting gap is correlated with the characteristic energy, Γ, of the short-range spin fluctuation.

5. The anomalous transport properties in underdoped cuprates

The retarded and advanced in-plane Green's functions $G_{R,A}(p,\varepsilon)$ of the hole are $1/(\varepsilon-\xi_p\pm {\rm i}\Gamma(p,\varepsilon))$, where $\xi_p=(p^2-k_{\rm F})/2m^*$, $k_{\rm F}$ is the Fermi momentum, and m^* is the effective mass of the hole, $\Gamma(p,\varepsilon)$ is the scattering rate, and contains some relaxations. First, the relaxation due to massless $U(1)$ gauge field A_μ^3 [26] is

$$\tau^{-1}(\varepsilon,p)\propto T\int {\rm d}p'^2\,{\rm d}\omega\,{\rm Im}\,G_{\rm N}(\varepsilon-\omega,p')V_\mu(p)\,{\rm Im}\left[\left(\delta_{\mu\nu}-\frac{q_\mu q_\nu}{q^2}\right)\times(\omega^2-c^2q^2+{\rm i}/2\tau_{\rm inst})^{-1}\right]V_\nu(p)(1-\coth\theta_{p,p'})$$

where $G_{\rm N}(r,\tau)$ are N_a propagators, $\theta_{p,p'}$ is the scattering angle and $V_\mu(p)$ is the vertex. $\tau_{\rm inst}$ is the relaxation time of the gauge field A_μ^3 by the instanton-like spin fluctuation [27]. This relaxation contributes dominantly to the T-linear resistivity in the normal state [26].

The second relaxation is due to emission and adsorption of the massive gauge fields A_μ^1 and A_μ^2.

From equation (3), we can obtain the Green function of the massive gauge fields A_μ^1 and A_μ^2 in the 't Hooft–Feynman gauge as follows, that is, the Fourier transform of $\langle A_\mu^a A_\nu^a\rangle_{a=1,2}$ is $D_R(k,\omega)\sim g_{\mu\nu}\big/\left(\omega^2-\left(k^2+{m_1}^2\right)+\Pi\right)$.

According to the Keldysh method [28–32], the hole energy relaxation time τ_ε is defined by the following kinetic equation,

$$\frac{1}{\tau_\varepsilon}\propto-\frac{\delta}{\delta n_\varepsilon}\int {\rm d}p\frac{1}{(2\pi)^3}\,{\rm Im}[G^A(p,\varepsilon)]\Gamma(p,\varepsilon) \quad (11)$$

where n_ε is the hole energy distribution function. In the random-phase approximation, the collision integral, $\Gamma(p,\varepsilon)$, is represented as follows,

$$\Gamma(p,\varepsilon)=-2\iint {\rm d}q\,{\rm d}\omega\frac{1}{(2\pi)^4}\,{\rm Im}[G^A(p+q,\varepsilon+\omega)]{g_2}^2\,[{\rm Im}\,D_R(q,\omega)]\,R_T(p,q,\varepsilon,\omega) \quad (12)$$

$$R_T=[1+N_T(\omega)]f(p+q,\varepsilon+\omega)[1-f(p,\omega)]-N_T(\omega)[1-f(p+q,\varepsilon+\omega)]f(p,\omega) \quad (13)$$

where $N_T(\omega) = -\frac{1}{2}\left[1 - \coth\frac{\omega}{2T}\right]$, and f is the hole distribution function.

Then the collision integral $\Gamma(p, \varepsilon)$ equals

$$\Gamma(p,\omega) = -2\iint \mathrm{d}p\,\mathrm{d}\omega \frac{1}{(2\pi)^4}\,\mathrm{Im}\,[G^A(p+q,\varepsilon+\omega)]{g_2}^2 \times \frac{\mathrm{Im}\,\Pi}{\left(\omega^2-\left(q^2+{m_1}^2\right)+\mathrm{Re}\,\Pi\right)^2+(\mathrm{Im}\,\Pi)^2} R_T(p,q,\varepsilon,\omega). \tag{14}$$

Since the important energy region of the hole is around the Fermi energy, we can consider the collision integral in the condition of $\varepsilon \sim \omega \ll \sqrt{q^2+m_1^2}$. In addition, taking account of $\mathrm{Im}\,\Pi \sim m_1 < \sqrt{q^2+m_1^2}$, the collision integral $\Gamma(p,\varepsilon)$ is represented approximately as

$$\Gamma(p,\varepsilon) = -2\iint \mathrm{d}q\,\mathrm{d}\omega \frac{1}{(2\pi)^4}\,\mathrm{Im}\,[G^A(p+q,\varepsilon+\omega)]{g_2}^2\,\mathrm{Im}\,\Pi\, R_T(p,q,\varepsilon,\omega)$$
$$\propto m_1(p){g_2}^2 \iint \mathrm{d}q\,\mathrm{d}\omega\,\mathrm{Im}\,[G^A(p+q,\varepsilon+\omega)]R_T(p,q,\varepsilon,\omega). \tag{15}$$

Now we shall discuss the collision integral $\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)$ of the Fermi momentum $\hat{k}_{\mathrm{F}}$. $I(\hat{k}_{\mathrm{F}},\varepsilon)$ equals

$$\Gamma(\hat{k}_{\mathrm{F}},\varepsilon) \propto m_1(\hat{k}_{\mathrm{F}}){g_2}^2 \iint \mathrm{d}q\,\mathrm{d}\omega\,\mathrm{Im}[G^A(\hat{k}_{\mathrm{F}}+q,\varepsilon+\omega)]R_T(\hat{k}_{\mathrm{F}},q,\varepsilon,\omega)$$
$$\sim \mu(\hat{k}_{\mathrm{F}})g_4{g_2}^2 \iint \mathrm{d}q\,\mathrm{d}\omega\,\mathrm{Im}[G^A(\hat{k}_{\mathrm{F}}+q,\varepsilon+\omega)]R_T(\hat{k}_{\mathrm{F}},q,\varepsilon,\omega). \tag{16}$$

Since the value, $\mu(\hat{k}_{\mathrm{F}})$, is much higher around the hot spot, the value of the collision integral $\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)$ for the hole of the Fermi momentum $\hat{k}_{\mathrm{F}}$ near $(\pi, 0)$ or $(0, \pi)$ becomes higher. This implies that the hole lifetime near the $(\pi, 0)$ or $(0, \pi)$ is unusually short.

On the other hand, the value, $\mu(\hat{k}_{\mathrm{F}})$, is much reduced around the cold spot. As a result, the collision integral $\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)$ for the hole of the Fermi momentum $\hat{k}_{\mathrm{F}}$ parallel to (π, π) decreases remarkably. This means that the hole lifetime in cold spots near the zone diagonal is much longer than elsewhere on the Fermi surface.

Now we shall consider the c-axis conductivity σ_c in the present theoretical formula. In a very anisotropic system such as underdoped cuprates, an adequate theoretical expression is

$$\sigma_c = \int \frac{\mathrm{d}^2 p}{(2\pi)^2} t_\perp(p)^2 G_R(p,\varepsilon)G_A(p,\varepsilon)\frac{\partial f}{\partial\varepsilon}. \tag{17}$$

Here $t_\perp(p)$ is the interplane hopping, $f(\varepsilon)$ is the Fermi function and $G_{R,A}(p,\varepsilon) = 1/(\varepsilon - \xi_p \pm \mathrm{i}\Gamma(p,\varepsilon))$ are the retarded and advanced in-plane Green's functions of the hole, where $\Gamma(p,\varepsilon)$ is shown in equation (12). The interplane hopping $t_\perp(p)$ is strongly momentum dependent, being very small for the Fermi momentum parallel to (π, π) and breaking maximal for the Fermi momentum parallel to $(\pi, 0)$ [33].

Thus the c-axis conductivity σ_c can be approximated as follows,

$$\sigma_c \propto \sum_{\hat{k}_{\mathrm{F}}} t_\perp(\hat{k}_{\mathrm{F}})^2 \frac{1}{\left(\varepsilon-\xi_{\hat{k}_{\mathrm{F}}}\right)^2+\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)^2}\frac{\partial f}{\partial\varepsilon} \tag{18}$$

where $\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)$ is given in equation (16). The value of $t_\perp(k_{\mathrm{F}},\varepsilon)$ is much higher around the hot spot. Therefore, contributions due to the $\Gamma(\hat{k}_{\mathrm{F}},\varepsilon)$ around the hot spot dominate the c-axis conductivity. Recently Lavrov *et al* [34] have found the very interesting result that the c-axis conductivity is reduced remarkably upon cooling through the Néel temperature T_{N}. The present theory can explain naturally this experimental result.

That is, the growth of the antiferromagnetic state, upon cooling through the Néel temperature, increases remarkably the distortion $\mu\left(\hat{k}_{\mathrm{F}}^h\right)$ around the doped hole at the hot spot.

Taking into account equation (16) and (18), the increase of $\mu\left(\hat{k}_{\mathrm{F}}^{h}\right)$ induces a reduction of the c-axis conductivity.

6. Conclusion

We have given some explanations for the broad spectra around $(\pi, 0)$ of ARPES, short-range spin fluctuation and anomalous transport properties in underdoped cuprates, by using quantized massive collective gauge fields around the doped hole.

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